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FINAL TECHNICAL REPORT

T.H. Lin
University of California, Los Angeles

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Scientific Officer: Yapa Rajapakse
P.I. and Affiliation: Professor T.H. Lin, Professor
Department of Civil Engineering
University of California, Los Angeles
Los Angeles, CA 90024-1593

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TABLE OF CONTENTS

I. Introduction
- a) Dependency of Slip on Resolved Shear Stress
- b) Previous Theories

II. Present Micromechanic Theory
- a) Analogy of Inelastic Strain and Applied Forces
- b) Initial Stress Field
- c) Gating Mechanism provided by Stress Field
- d) Supporting Experimental Observations

III. The main accomplishments of this research are as follows:
- a) Fatigue crack initiation under cyclic torsion:
- b) Fatigue Crack Initiation under Combined Cyclic Axial Loading and Torsion:
- c) Interaction of two slip planes in fatigue crack initiation:
- d) Interaction of Fatigue and Creep in High-Cycle Fatigue Crack Initiation:
- e) Overload Effect on the Retardation of Fatigue Crack Initiation:
- f) Fatigue Crack Initiation in Ordered Alloys:

IV. Index of technical reports published:

V. Index of all publications:

VI. References

I. Introduction

More than 90% of all catastrophic ruptures occurring in practice are caused by fatigue of materials [1]. Fatigue crack occurs in two stages (1) fatigue crack initiation and (2) fatigue crack propagation. Crack initiation dominates the life of high-cycle fatigue and is also a necessary step for crack propagation. Hence the study of crack initiation is of highly practical importance. This research is to develop a micromechanic theory of high-cycle fatigue crack initiation. This problem is a problem of long standing, has been studied by many distinguished scientists including N.F. Mott and is of great scientific interest.

As pointed out by Kennedy, 1961 [2], a large amount of test data is available. The difficulty seems to see these multitudinous facts as a related and connected whole, largely because of the lack of a sound general theory and also because of the very great complexity of metallurgical effects. With the development of dislocation theory since about 1940, new explanations of the fatigue effects have been put forward. Dislocation explains the characteristics of metals at the atomic level, which is of course, very important. However, dislocations may be too fine to correlate even the macroscopic phenomena observed under optical microscope, such as fatigue bands [2]. Hence a micromechanic approach is here adopted.

McCommon and Rosenberg [3] and MacCone et al. [4] showed that metals are subject to failure at temperatures as low as 1.7°K. This indicates that although surface corrosion, gas adsorption, gas diffusion into the metal and vacancy diffusion to form voids can have important effect on fatigue, but they are not necessary to fatigue failure. This seems to leave mechanics, i.e., the local stress and strain, as a basic mechanism of fatigue.

a) Dependency of Slip on Resolved Shear Stress

Single crystal tests [5,6] have shown that under stress, slip occurs along certain crystal directions on certain planes. Slip depends on the shear stress along this direction on this plane, called resolved shear stress and is independent of the normal stress on the sliding plane. The resolved shear stress to initiate or to cause the continuation of slip is called the critical shear stress. This dependency of slip on the resolved shear stress under monotonic loadings has been found to hold also under cyclic loadings [7].

b) Previous Theories

Forsyth and Stubbington 1954 [8] reported the detection of extrusion in slip bands during fatigue of some aluminum alloys. Thompson, Wadsworth and Louat [9] and Hull [10] detected the extrusion process in both copper and aluminum. This initiation of extrusion process was also observed by Meke and Blochwitz [11] and Mughrabi [12] in their studies of persistent slip bands.

Following the clue, which the observations on extrusions and intrusions in slip bands have provided, a number of theories of fatigue crack initiation have been proposed by different very distinguished investigators. For example, Mott [13] proposed that a screw dislocation repeats its path through cross slip. He considered a column of metal containing a single screw dislocation intersecting a free surface. When the dislocation travels a complete circuit, the volume contained in the circuit is translated parallel to the dislocation, this causes the metal to extrude. This mechanism does not explain why the dislocation under cyclic stressing, does not oscillate back and forth along the same path rather than traversing a closed circuit. Clearly some form of gating mechanism is required to convert the back and forth oscillations into unidirectional circuits. Cottrell and Hull [14] proposed that Frank Read sources exist on two intersecting slip planes and a complete cycle of forward and reversed loading results in an extrusion and intrusion. Such a model would predict the extrusion and intrusion to form in neighboring slip bands and to be inclined to each other, but they have been found to occur in the same slip band and to be parallel to each other. Wood [15] proposed a simple model of a single operative slip system. An unidirectional stressing causes layers of metal to slide in the same direction: but forward and reverse stressing causes different amounts of net slip on different planes and results in peaks and valleys. However, this model does not explain why, under an alternate loading, the slip continue to monotonically deepen the valley and raise the peaks as observed in experiments. Drawbacks of other theories have also been discussed by Kennedy [2].

For a dislocation to glide, it (1) must glide along a certain direction on a certain crystal plane and (2) must subject to a resolved shear stress equal to or greater than the critical shear stress. The above mentioned theories show the possible paths of dislocation movement to satisfy condition (1) but the resolved shear stress field caused by the dislocation movement that has significant effect on (2) was not considered. In the present study, this important effect of this stress field, which supplies a natural gating mechanism, is shown.

II. Present Micromechanic Theory

When the resolved shear stress in a slip system in some region of a polycrystal reaches the critical shear stress, slip can occur in this system. If the loading on the metal is removed, this slip remains and causes a residual stress field. If a different loading is then applied, the resolved shear stress is the sum of the residual and the applied stresses. In calculating this residual stress field, the analogy between the inelastic strain and the applied force, developed by Lin, 1968 is here used. The analogy is briefly reviewed here.

a) Analogy of Inelastic Strain and Applied Forces

Referring to a set of rectangular coordinates, the strain component is composed of the elastic part denoted by single prime and the inelastic part denoted by double prime

$$e_{ij} = e'_{ij} + e''_{ij}, \tag{1}$$

Thermal, creep and plastic strains are considered to be the inelastic strain. Neglecting the anisotropy of elastic constants, the stress is related to the elastic strain as

$$\begin{aligned} \tau_{ij} &= \delta_{ij}\lambda\Theta' + 2\mu e'_{ij}, \\ \tau_{ij} &= (\delta_{ij}\lambda(\Theta - \Theta'') + 2\mu(e_{ij} - e''_{ij}) \end{aligned} \tag{2}$$

where λ and μ are Lame's constants, Θ is the dilatation. The condition of equilibrium within a body of volume V is

$$\tau_{ij,j} + F_i \quad = \quad 0 \text{ in } V, \tag{3}$$

where the subscript after the comma denotes differentiation, the repetition of the subscript denotes summation from one to three, and F_i denotes the body force per unit volume along the x_i axis. At any point on the boundary Γ with normal ν, the i-component of the surface traction per unit area $S_i^{(\nu)}$, can be written from the condition of equilibrium as

$$S_i^{(\nu)} \quad = \quad \tau_{ij}\nu_j \text{ on } \Gamma \tag{4}$$

where ν_j is the cosine of the angle between the normal ν and the x_i axis. Substituting (2) into (3) and (4), we obtain

$$\delta_{ij}\lambda\Theta_{,j} + 2\mu e_{ij,j} - (\delta_{ij}\lambda\Theta''_{,j} + 2\mu e''_{ij,j}) + F_i = 0 \tag{5}$$

$$S_i^{(\nu)} \quad = \quad \nu_j[\delta_{ij}\lambda\Theta + 2\mu e_{ij} - (\delta_{ij}\lambda\Theta'' + 2\mu e''_{ij})] \tag{6}$$

It is seen that - $(\delta_{ij}\lambda\Theta''_{ij} + 2\mu e''_{ij,j})$ and $(\delta_{ij}\lambda\Theta'' + 2\mu e''_{ij})\nu_j$are equivalent to F_i and $S_i^{(\nu)}$ in causing the strain field e_{ij}, and are here denoted by $\overline{F}_i$ and $\overline{S}_i^{(\nu)}$, respectively, giving.

$$\delta_{ij}\lambda\Theta j + 2\mu e_{ij,j} + F_i + \overline{F}_i \quad = \quad 0 \tag{7}$$

$$S_i^{(\nu)} + \overline{S}_i^{(\nu)} \quad = \quad \nu_j(\delta_{ij}\lambda\Theta + 2\mu e_{ij}). \tag{8}$$

Hence, the strain distribution in a body with inelastic strain under external load is the same as that in an elastic body (no inelastic strain) with the additional equivalent body and surface forces $\overline{F}_i$ and $\overline{S}_i^{(\nu)}$. This reduces the solution of stress field of a body with known elastic strain distribution to the solution of an identical elastic body with an additional set of equivalent body and surface forces. This gives the same results as the famous process of imaginary cutting, relaxing, restoring, welding, and relieving in the noteworthy paper by Eshelby, 1957, on ellipsoidal inclusions [16].

If the inelastic strain is due to thermal strain alone with thermal coefficient of expansion α and temperature T, we can write

$$e''_{ij} = \delta_{ij}\alpha T \ , \quad \Theta'' = e''_{ii} = 3\alpha T$$

Then the equivalent body and surface forces becomes

$$\overline{F}_i = -(2\lambda + 2\mu)\,\alpha\, T_{,i}$$

$$\overline{S}_i^{(\nu)} = \nu_i(3\lambda + 2\mu)\,\alpha\, T$$

This is the well-known Duhammel's analogy [17], between temperature gradient and the body force in an elastic medium. Hence Duhammel's analogy is a special case of the general analogy for inelastic strain.

This analogy has been used by Lin [18] in the derivation of the macroscopic polycrystal stress-strain and stress-strain-time relations under radial and non-radial loadings from those relations of the component crystals.

b) Initial Stress Field

Imperfections like dislocations exist in all metals and cause initial stress fields. For a slice of metal to extrude out of a surface, positive shear deformation must occur on one side of the extrusion and negative shear on the other. The initial stress field τ^I near the free surface favorable for the initiation of extrusion is one with positive resolved shear stress above the slice and a negative one below the slice. Referring to Fig. 1, x_1 and x_2 are a set of rectangular axes on a longitudinal section of polycrystalline metal subject to alternate tension and compression along x_2 - axis. α and β are another set of rectangular axes with β along the slip direction and α along the normal to the slip plane of the most favorably-oriented crystal at the free surface. Both α and β make 45° with x_1 and x_2 axes. From the analogy of applied force and plastic strain as discussed previously, an initial stress field caused by a linear variation of $e''_{\alpha\alpha}$ from zero at the free surface to maximum at the interior boundary of this crystal has been calculated [19] and is found to give a positive shear stress in P, and negative in Q. This initial shear stress field clearly is favorable for the initiation of an extrusion.

Consider a perfect crystal. If we cut a slit through this crystal and force a sheet of metal of one atom thick into the slit, a pair of parallel edge dislocation A and B of opposite signs forming an interstial dipoles is produced as shown in Fig. 2. If we cut a rectangular block along the dotted line, the free length of this block will be one atomic spacing more than the corresponding length of the hole. If there are n such dipoles in a length of N atomic spacings, this will give an initial strain $e^{I}_{\alpha\alpha}$ of n/N. Hence this initial strain can be caused by an array of dislocation dipoles. This array of dipoles was suggested by Lin and Ito [20] in 1969 as a possible way of providing the initial strain to cause the favorable initial stress field. Recently these dislocation dipoles were observed in fatigue specimens as ladder structures in persistent slip bands [21].

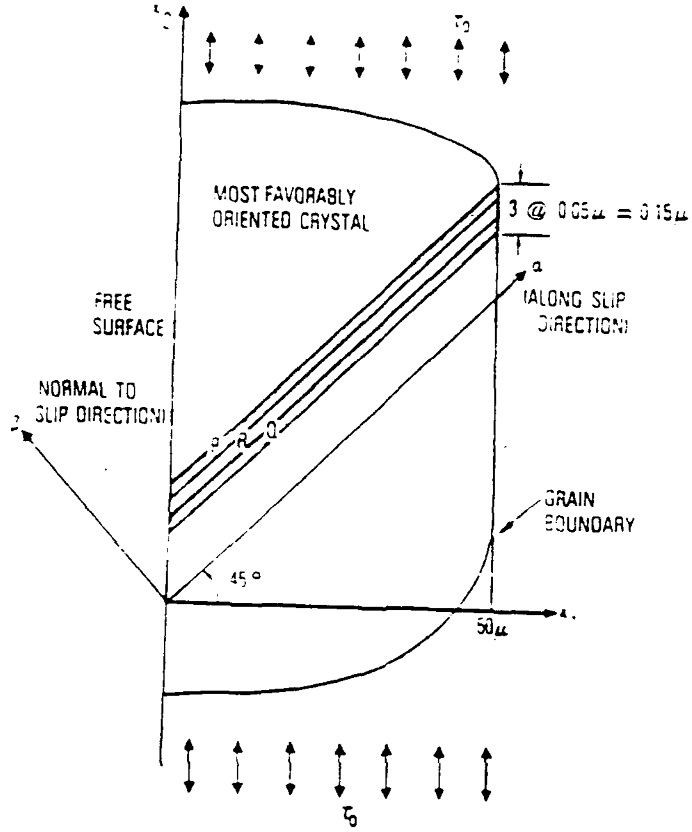

Fig. 1. Most favorably oriented crystal at free surface

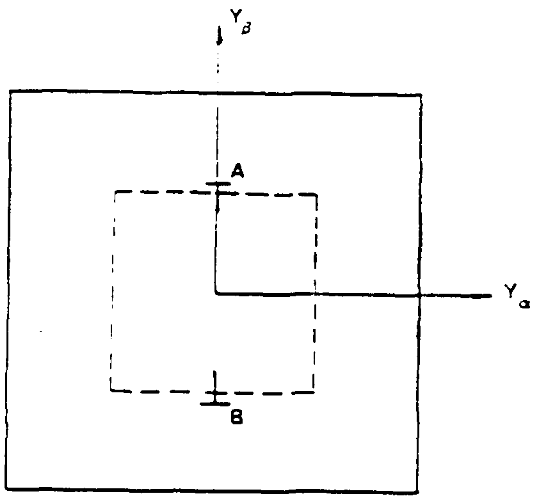

Fig. 2. A dislocation interstitial dipole

c) Gating Mechanism provided by Stress Field [22,23]

A tensile loading causes a positive resolved shear stress τ^A in the whole crystal. In P the resolved shear stress will be the sum of τ^I and τ^A. This stress will be the first to reach the critical shear stress τ_c to cause slip. This slip causes a residual stress τ^R. Due to the continuity of the stress field τ^R, slip in P relieves not only the positive shear stress in P, but also in its neighboring region including Q. This keeps the positive shear stress in the neighboring region from reaching that of P during the forward loading. Hence only P slides in the forward loading. The relief of positive resolved shear stress has the same effect as increasing negative resolved shear stress. During the reversed loading, Q has the highest negative resolved shear stress and hence slides. This slip causes the relief of negative shear stress not only in Q but also in its neighboring region including P. This relief of negative resolved shear stress has the same effect as increasing positive resolved shear stress, thus causing P to be more ready to slide in the next forward loading. During the next forward loading, P has the highest positive shear stress and hence slides. This slip again relieves the positive shear stress and increases the negative shear stress in Q thus causing it to slide in the next reversed loading. This process is repeated and gives the natural gating mechanism to cause alternate sliding in P and Q. As a result, positive slip in P and negative slip in Q increase monotonically with cycles of loading and produce an extrusion. The interchange of the signs of the initial resolved shear stresses in P and Q will yield an intrusion instead of an extrusion. This theory explains the observed monotonic raising of extrusions and deepening of intrusions, and also shows that such an initial stress field can be obtained by a given plastic strain distribution which can be caused by a distribution of dipoles [19] in the region between P and Q. Same mechanism also exists in cyclic torsion.

d) Supporting Experimental Observations

The above theory has many experimental evidences. Few of these are listed below.

1. Formation of new slip lines in reversed loading. Tests on single aluminum crystals under cyclic loading in tension and compression by Charsley and Thompson [24], have shown that

a reversal of stress after a prior forward deformation gives rise to new parallel slip lines. Buckley and Entwistle [25] found that on an aluminum crystal, slip lines formed during compressive loading lie between those formed in prior tensile loading. These and other tests show the occurrence of slip lines in the reversed loading to be very close but distinct from those formed in forward loading just like P and Q in the proposed theory.

2. Gough [26] tested two single crystals in cyclic torsion with superimposed static tensile load. The test aims to determine whether the maximum shear stress or the maximum range of shear stress determines slip under alternate loading. The maximum shear stress in this case acted on a plane different from that with maximum range of shear stress. It was found that the maximum shear stress determined the slip system only in the very early stages of the test but very soon the slip changed to the slip system with maximum range of stress. This agrees with the present theory since it shows the dependence of the build-up of plastic strain on the range of stress.

3. X-ray reflection patterns of monotonically and cyclically loaded specimens are very different [15]. The latter retain the discrete spots like that of annealed metals while the former do not, Fig. 3. This shows that slip occurrence in alternate loadings does not cause lattice straining in the bulk of the metal. Under cyclic loading, the positive shear slip line (like P) are closely located with the negative ones (like Q). At some distance from the slip lines, the stress field caused by positive slip in P is balanced by that caused by negative slip in Q. Hence, the stress field and the lattice strain is small in the bulk of the metal. Under monotonic loadings, the slip in all slip lines tends to be all of the same sign and causes a significant average plastic strain which causes an appreciable stress field and elastic lattice strain in the bulk of the metal. The above theory accounts for the different X-ray reflection patterns of the monotonically and cyclically deformed metals.

4. Wood and Bender [27], tested copper circular rod specimens subject to torsion. The specimens were electropolished and then scratched as markers with a pad carrying 0.5μ diamond dust. Some specimens were subject to alternate torsion and some subject to single twist through large angles. The deformation in a typical slip band *AB* of a specimen subject to a single twist is shown in Fig. 4; *a*, *b*, *c* are typical scratches which were initially straight and continuous. It is seen that the single twist caused the scratches above *AB* to displace relatively to those below. Fig. 5 shows the deformation under cyclic torsion with scratch *d*, *e*, *f* and a typical fatigue band *DC*. It is seen that the cyclic deformation caused no relative displacement of the scratches left and right of the fatigue bands, but within the band, the scratches have displaced equally up and down producing a zigzag. A severely slid line with positive shear such as *P* is sandwiched by two less severely slid lines with negative shear such as *Q*. This clearly verifies with the theory proposed.

Figure 3. (a) **Sharp X-Ray Reflection from Annealed α-Brass.**
(b) **From Same Specimen as (a) After a Unidirectional Strain 150 × 0.5° Twist.**
(c) **From Same Specimen as (a) After 1500 Reversals of Plastic Strain 0.5° Twist and Showing Same Reflection as (a).**

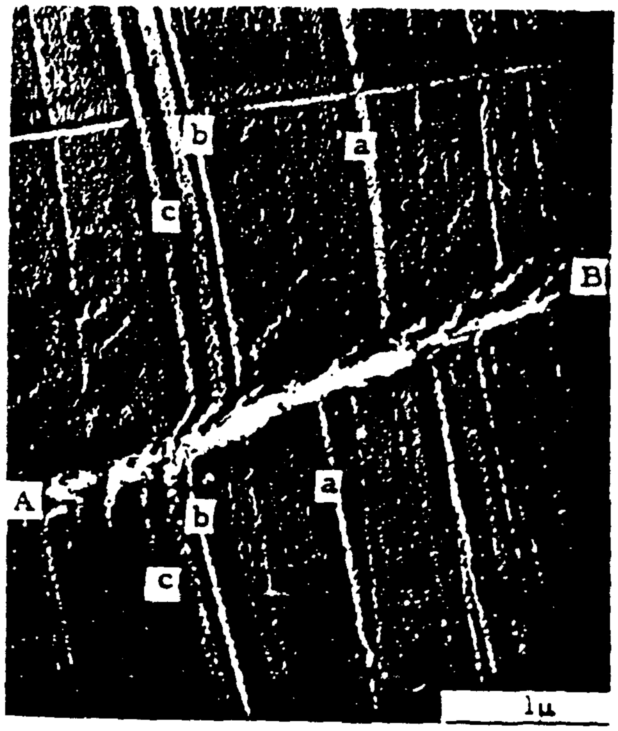

Fig. 4. Initially Straight Scratches a, b, c are displaced unidirectionally by static slip band AB.

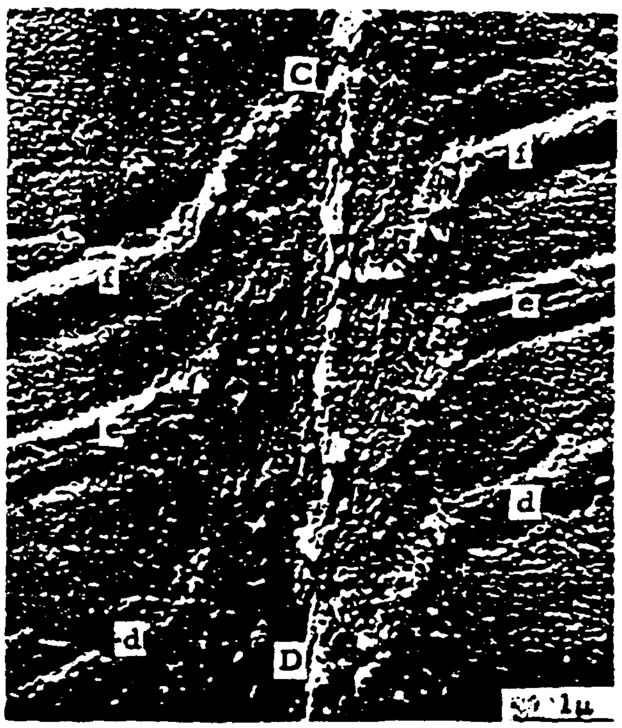

Fig. 5. Cyclic Slip Band CD Produces no Overall displacement of Scratches d, e, f, within the slip band the scratches are displaced equally backward and forward

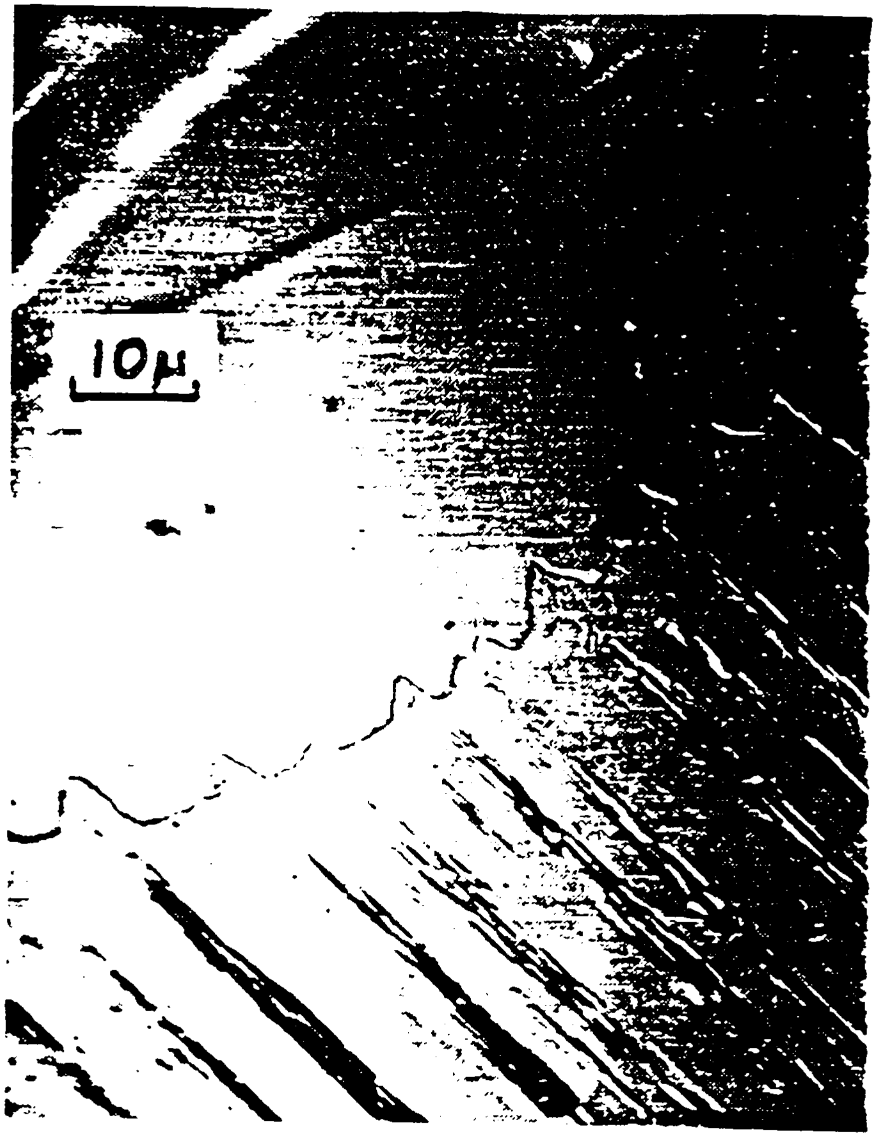

Fig. 6. Grain Boundary at the End of Slip Bands in Fatigued Aluminum.

5. Forsyth [28] has given a picture showing the moving of grain boundary at the end of a slip band. It is seen in Fig. 6 that snears of opposite signs are closely associated.

6. Recent tests by Woods [29] of single copper crystals have shown that from the very early stages of fatigue tests the specimens develop into a state containing two phases: a soft phase with persistent slip bands into which the deformation tends to concentrate, and a hard phase with almost inactive matrix which is comparatively dislocation-free. The regions in P and Q slices of the present model correspond to the observed soft-phased and those outside of P and Q correspond to the hard phased regions.

7. Recently Meke and Blochwitz [11] have indicated that persistent slip band protrudes out in two sides of a single crystal under cyclic loading. If the initial stress varies from positive to negative in P and from negative to positive in Q, cyclic loading will cause extrusions on both faces of the single crystal. The movement of the subgrain boundaries Fig. 11 as shown by Meke and Blochwitz seem to agree well with this proposed theory.

Using the amount of local plastic strain in P as an estimate of the early fatigue damage [30], this theory has been used to calculate the effects of mean stress [31], grain size and strain hardening [32] on this fatigue damage under cyclic tensile and compressible loadings. The calculated results seem to agree, in general, with test results.

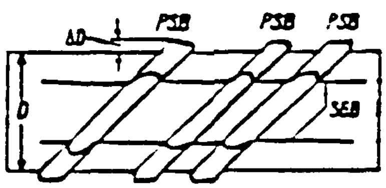

Fig. 7. Extruding on Two Opposite Surfaces of a single crystal. PSB Persistent Slip Band SGB Subgrain Boundary Taken from Ref. 11

The above explains the deficiencies of the previous theories and shows how the present approach removes these deficiencies and agree with experimental including microscopic metallurgical observations.

III. The main accomplishments of this research

a) Fatigue crack initiation under cyclic torsion:

Single crystal tests have shown that extrusion and intrusion occurred on a slip plane along the most highly stressed slip direction and did not occur when the direction was parallel to the free surface. In cyclic tension and compression, the slip direction of the highest stressed slip system in the most favorably oriented crystal inclines 45° with the free surface. In a circular shaft under torsion, the maximum shear stress occurs along the circumferential direction (parallel to the free surface) on a plane normal to the shaft axis. Hence, the extrusion and intrusion process will not occur in the slip system with maximum shear stress. But this process may occur on some crystal with a slip plane normal to the shaft and a slip direction making angle β with the circular boundary. Two thin slices spaced at $0.1\mu m$ apart were considered. The top slice is assumed to have a positive initial resolved shear stress and the bottom one, a negative shear stress. The microstress fields caused by plastic strain in these two slices were calculated and shown to provide the gating mechanism for the two slices to slide alternately just as in the case of cyclic tension and compression as developed by Cooley and Lin, 1986 [33]. Lin et al., 1987 [34]. Displacement component normal to the free surface is taken as a measure of the amount of intrusion, which in turn, is taken as a measure of crack initiation. It was found that the angle β, giving this maximum displacement component increases with the cyclic torsional stress. Under the same initial shear stress, and the same range of shear stress with zero mean stress, the growth of the crack initiation is much larger in cyclic tension and compression than in cyclic torsion for small range of shear stress. However, as the amplitude of the range of shear stress is increased, the rate of crack initiation for cyclic torsion does catch up with that of cyclic tension and compression. This agrees with test results reported by Tanaka et al. in their paper "Fatigue Strength 7075-T6-AL.AL. under Combined Axial and Torsional Loadings", published in Fatigue Eng'g Material & Structure, Vol. 7, p. 195, 1984. The detail of the present work is shown in the paper "Fatigue Crack Initiation under Cyclic Torsion", published in the Journal of Applied Mechanics Vol. 53 p. 550-554 1986.

b) Fatigue Crack Initiation under Combined Cyclic Axial Loading and Torsion:

Under combined axial loading and torsion on a circular shaft, the stress on the outer layer of the shaft is under a combined axial and shear stresses in a plane. The principal planes giving extreme value of normal stresses are shown by the dotted line Fig. 8. The planes giving maximum shear stresses are making 45° with the principal planes. Now we have two planes of maximum shear stresses. One is $\frac{\sigma_1}{2}$ making 45° with *xy*-plane causing a fatigue crack initiation same as that caused by pure axial loading.

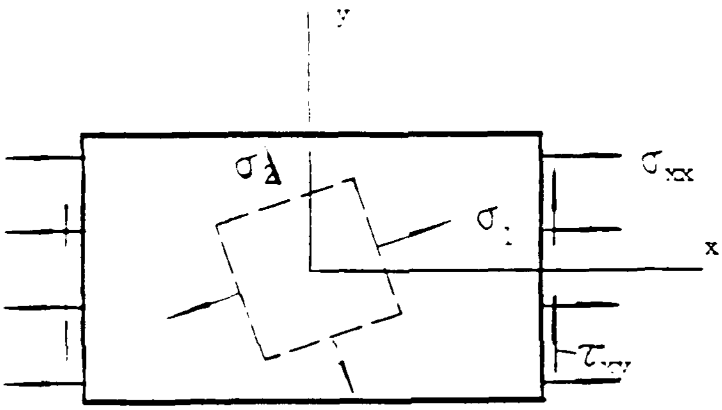

Fig. 8. Combined Axial and Shear Loading

Due to the presence of torsional stress τ_{xy}, the other one is on a plane making 45° with the two principal planes.

$$\tau_{max} = \sqrt{(\tau_{xy}/2)^2 + \tau_{xy}^2}$$

The later is treated like the shear stress due to pure torsion. Among these two crystals, the one with higher crack initiation rate gives the crack initiation rate of the polycrystal. This work was presented in the 3rd International Conference on Fatigue and Fatigue Thresholds and was very well received. This work was published in the proceedings of this conference, (1987).

c) Interaction of two slip planes in fatigue crack initiation:

Referring to the figure 2, for a slice R to extrude out of the free surface, positive shear deformation must occur in P and negative in Q. The initial shear stress field favorable to this deformation is one positive in P and negative in Q. These initial shear stress can be caused by an inelastic tensile strain $e^{I}_{\alpha\alpha}$ in R, which can be caused by an array of interstitial dislocation dipoles. These dipoles have recently been observed in slip bands as ladder structure in fatigue specimens and produce an initial tensile strain along the extrusion direction in R. If R were cut out from the metal, the unconstrained length of R would be longer than the slot in the metal by an amount, which Mughrabi called "static extrusion" [35]. If it were forced back into the slot in the metal, R would be subjected to a compressive longitudinal stress $\sigma_{\alpha\alpha}$, which causes the positive and negative initial shear stress $\tau_{\alpha\beta}$ in P and Q. Under cyclic loading, plastic shear strain $\gamma''_{\alpha\beta}$ builds up in P, Q. This $\gamma''_{\alpha\beta}$ in P is about the same as in Q but $\gamma''_{\alpha\beta}$ varies along α-direction. Let "t" denote the thickness of slices P, Q. The displacement along α-direction in R is then $t\gamma''_{\alpha\beta}$. It's differentiation with respect to α gives a tensile strain in R. This tensile strain relieves some initial compressive stress in R. Further cyclic loading causes tensile stress in some part of R. This tensile stress causes a resolved shear stress in a second slip system. When this shear stress reaches critical, the second sliding plane slides. This slip causes plastic tensile strain, which has the same effect as the initial tensile strain. Hence, this secondary slip helps P, Q to slide and causes the extrusion to grow beyond the "static extrusion" as reported by Mughrabi [35] in his experimental study of fatigue. The detail of this study is shown in the paper on "Micromechanics of an Extrusion in High-Cycle Fatigue" [36] by T.H. Lin, S.R. Lin and X.Q. Wu published in Philosophical Magazine A, Vol. 59, pp. 1263-1276, 1989.

d) Interaction of Fatigue and Creep in High-Cycle Fatigue Crack Initiation:

Single crystal tests at room and elevated temperatures below one half of the melting temperature, have shown that under stress, slip occurs along certain directions on certain planes. This slip depends on the resolved shear stress. This dependency, known as Schmid's Law, holds also under cyclic loadings. An aluminum polycrystal under a cyclic loading in tension and compression at an elevated temperature is considered. Extrusions and intrusions are preferable sites of fatigue cracks. The extent of extrusions or intrusions are taken as a measure of fatigue crack initiation.

Single aluminum crystal tests at an elevated temperature has shown that the stress-strain-time relation can be approximately represented by the expression

$$\dot{e}^{c}_{\alpha\beta} = A(\tau_{\alpha\beta} - \tau^{c})$$

where $\dot{e}^{c}_{\alpha\beta}$ is the resolved shear strain rate, $\tau_{\alpha\beta}$ the resolved shear stress, τ^{c} is the critical shear stress and A is a constant.

Consider a most favorably oriented crystal at a free surface of the polycrystal as shown in Fig. 1. For an extrusion to start in a thin slice R sandwiched between two slices P and Q, positive shear must occur in P and negative in Q. An initial stress field to cause such sliding can be provided by an initial tensile strain $e^{I}_{\alpha\alpha}$ in R. The repetition of Greek subscript does not denote summation. This initial tensile strain can be provided by a row of interstitial dislocation dipoles and a negative $e^{I}_{\alpha\alpha}$ by vacancy dipoles.

A tensile loading τ_{22} on the polycrystal (Fig. 1) produces a positive τ^{A} in the whole crystal. Taking τ^{I} to be positive in P and negative in Q, we have $\tau^{I} + \tau^{A}$ in P reaching the critical shear stress τ^{c} first; and hence, P slides to yield creep strain $e^{c}_{\alpha\beta}$. Due to the continuity of stress field, slip in P relieves not only the positive shear stress in P but also in Q. Hence, this slip increases the negative resolved shear stress in Q to cause Q to slide more readily in the reverse loading. The negative slip in Q relieves the negative shear stress not only in Q, but also in P, thus causing P to slide more readily in the next forward loading. This process is repeated for every cycle thus providing a natural gating mechanism for a monotonic buildup of local slip strain $e^{c}_{\alpha\beta}$ in P and Q, pushing R out of the free surface and starts an extrusion. Interchanging the signs of the initial stresses in P and Q initiates an intrusion instead of an extrusion. This theory is extensively supported by metallurgical observations.

The initial tensile strain $e^{i}_{\alpha\alpha}$ in R causes the initial positive and negative shear stresses in P and Q. This, in turn, causes the growth of the extrusion. As the extrusion grows under cyclic loading, the slice R increases in length. This elongation causes the compression in R to decrease. A question has been raised as to whether the extrusion growth will cease after the extrusion has reached the static extrusion. The residual tensile stress $\tau^{R}_{\alpha\alpha}$ caused by elongation in R due to extrusion can cause changes of resolved shear stresses in all twelve slip systems. The resolved shear stress in one slip system may reach the critical and slide. The creep strain $e^{c}_{\xi\eta}$ caused by slip in this system has a tensor component $e^{c}_{\alpha\alpha}$ just like $e^{I}_{\alpha\alpha}$ in causing the positive and negative $\tau^{i}_{\alpha\beta}$, respectively, in P and Q. This secondary slip has been shown to increase greatly the extent of extrusion and intrusion in time-independent slip. The present study shows that this secondary slip also increases this extent of intrusion and extrusion in fatigue with creep. Creep strain under two different frequencies of loading were calculated. The results are published in the *Journal of Applied Mechanics*, Vol. 57, p. 807, Dec. 1990 [37].

e) Overload Effect on the Retardation of Fatigue Crack Initiation:

Copper single crystal tests by Hunsche and Neumann, 1986 have shown that under constant plastic strain amplitude of 0.20% the resolved shear stress increases from 32 MPa to 35 MPa within 500 cycles. The nucleation of persistent slip bands PSB is accompanied by a slight softening down to 32 MPa again. Then the shear stress amplitude stays constant. An overload causes more slip bands to become active. This softening after the peak stress causes a number of PSB's to continue sliding after the overload. A number of fatigue bands of different initial resolved shear stresses are assumed to exist in a most favorably oriented crystal at a free surface of a polycrystal. Without overload, only the band with the highest initial shear stress slides. With overload, many more PSB's become active and continue to slide after the overload. Slip in one PSB relieves some resolved shear stresses in other slip bands. The rate of slip in the band with highest slip rate decreases. The highest slip at the free surface is used as a measure of crack initiation. Hence overload generally causes retardation of fatigue crack initiation. Analytical method and numerical analysis have been made. The results were presented in Fatigue 90, the *4th International Conference on Fatigue and Fatigue Thresholds* held in July, 1990 in Hawaii, and published in its Proceedings, Vol. I, pp. 489-492, 1990 [38].

f) Fatigue Crack Initiation in Ordered Alloys:

Tests on Ni_3Al and Fe C-V ordered interatomic alloys show that long range order substantially increased the life in high cycle strain-controlled fatigue. Order favors planar slip by inhibiting cross-slip and/or multiple slip. Electron microscopic study of fatigue crack initiation in Ni_3Al single crystals by Hsiung and Stoloff shows that (111) primary slip bands first form. Then some of the bands become coarser. The coarse primary slip bands eventually developed into PSB-like bands. It was observed that cracks developed at PSB/matrix interface. The high-cycle fatigue crack initiation of disordered and ordered intermetallic alloys have been analyzed using the micromechanic theory. It has been found that the high-cycle fatigue crack initiation life of this ordered alloy is much longer than that of the disordered alloys. This explains the experimental results. The details of the analysis will be shown in the UCLA Engineering Report ENG-CE-91-01, Aug. 1991 [39].

IV. Index of technical reports published:

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"Micromechanics of an Extrusion in High-Cycle Fatigue with Creep", Lin, T.H., Lin, S.R. and X.Q. Wu, UCLA Engineering Report, Eng-89-09, Sept. 1989.

V. Index of all publications:

"Fatigue Crack Initiation under Cyclic Torsion", Cooley, W.U. and Lin, T.H., *Journal of Applied Mechanics*, Vol. 108, p. 550-554.

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